OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4753/01

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

QUESTION PAPER

TUESDAY 19 JUNE 2018: Afternoon

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book sent with the standard paper or any suitable paper supplied by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4753/01 sent with the standard paper
MEI Examination Formulae and Tables (MF2) sent with the standard paper
Insert for Question 2

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (SC/SW) 166170/1

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INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer ALL the questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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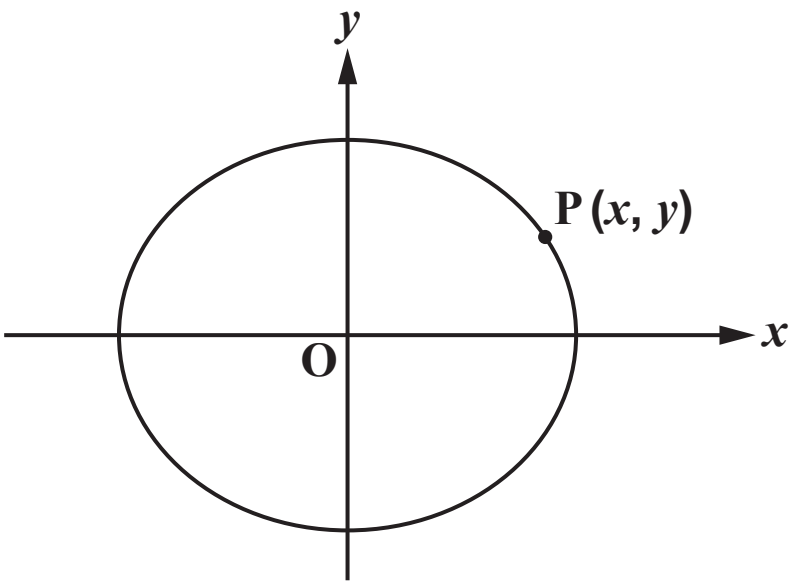
SECTION A (36 marks)

1 A point P moves round the curve with equation $3x^2 + 4y^2 = 4$. At time t, P is at the point (x, y), as shown in Fig. 1 below.

FIG. 1



(i) Find $\frac{dy}{dx}$ in terms of x and y. [2]

(ii) When P is at the point on the curve with x-coordinate 1 and positive y-coordinate, $\frac{dx}{dt} = 4$.

Find $\frac{dy}{dt}$ at this point. [4]

2 The three functions $\mathrm{f}(x)$, $\mathrm{g}(x)$ and $\mathrm{h}(x)$ are defined as follows:

$$\mathrm{f}(x) = \frac{x}{1 - 2x^2}, \quad \mathrm{g}(x) = 1 + \sin 2x \quad \text{and} \quad \mathrm{h}(x) = 3\mathrm{e}^{-2x^2}.$$

In the table in the Answer Book or in the table provided in the insert, write Yes or No in each space to indicate whether the function is odd, whether it is even, and whether it is periodic. If a function is periodic, state its period. [4]

3 The mass of a radioactive material decreases exponentially. Its 'half-life' is the time required for the mass of the material to reduce to half its initial value. The half-life of plutonium 241 is 14.4 years.

(i) Write down the percentage of the initial mass of plutonium 241 remaining after 28.8 years. [1]

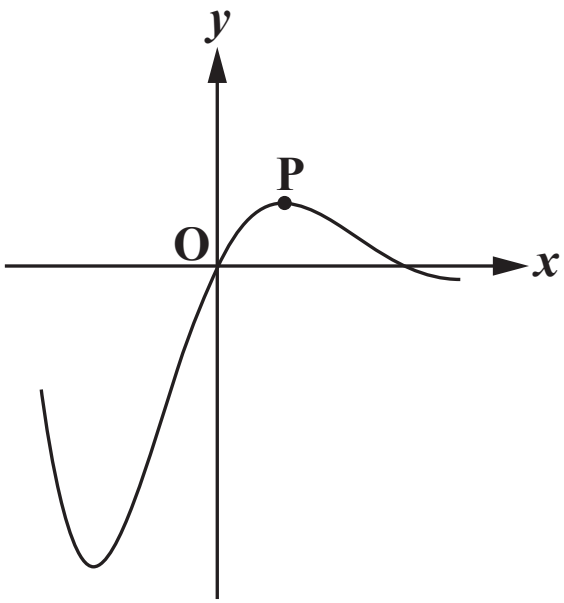
(ii) The mass M grams of plutonium 241 at time t years is given by the equation

$$M = M_0 \mathrm{e}^{-kt},$$

where M_0 grams is the initial mass and k is a constant. Find k, giving your answer correct to two significant figures. [3]

4 Fig. 4 below shows part of the curve with equation $y = e^{-x}\sin 2x$.

FIG. 4



Find the coordinates of the maximum point P. [6]

5 (i) On the same axes, sketch the graphs of $y = -|x + 1|$ and $y = 2x$. [3]

(ii) Solve the equation $-|x + 1| = 2x$. [2]

6 The function $h(x)$ is such that $h(x) = fg(x)$, where $f(x) = 2x + \frac{1}{2}\pi$ for $x \in \mathbb{R}$ and $g(x) = \arcsin x$ for $-1 \leqslant x \leqslant 1$.

(i) Find $h\left(\frac{1}{2}\right)$, giving your answer as a multiple of π. [2]

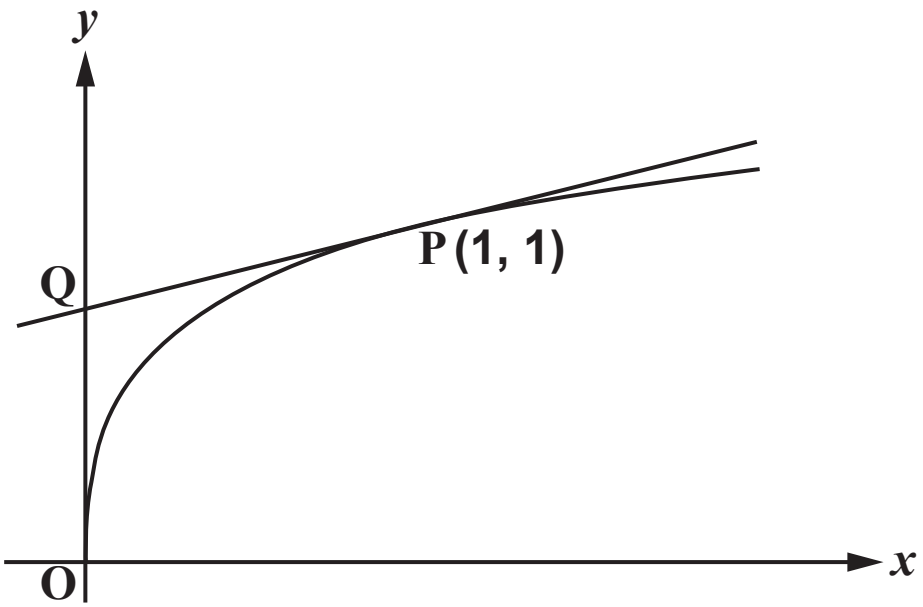
(ii) Find $h^{-1}(x)$. [4]

7 Prove that $n^3 - 3n^2 + 2n$ is divisible by 6 for all positive integers n. [5]

SECTION B (36 marks)

8 Fig. 8 shows the curve with equation $y = \dfrac{2\sqrt{x}}{1+\sqrt{x}}$. The tangent to the curve at P (1, 1) intersects the y-axis at Q.

FIG. 8



(i) Show that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x}(1+\sqrt{x})^2}$.

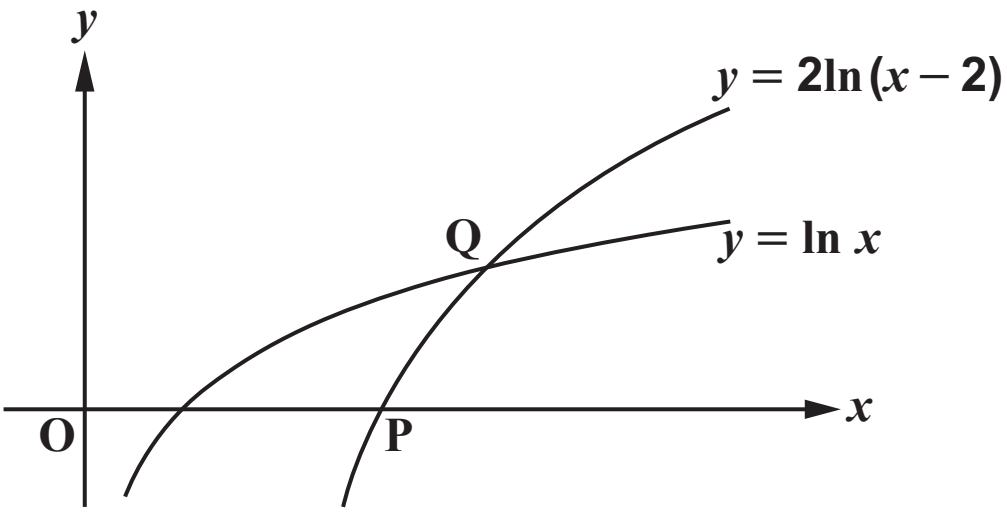
Hence find the equation of PQ, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [7]

(ii) Show that the substitution $u = 1 + \sqrt{x}$ transforms $\displaystyle\int \frac{2\sqrt{x}}{1+\sqrt{x}}\,dx$ to $\displaystyle\int \frac{4(u-1)^2}{u}\,du$. [3]

(iii) Hence find the exact area of the region enclosed by the curve, the y-axis and the line PQ. [8]

9 Fig. 9 below shows the curves with equations $y = \ln x$ and $y = 2\ln(x - 2)$ which intersect at Q. The curve $y = 2\ln(x - 2)$ crosses the x-axis at P.

FIG. 9



(i) Describe a sequence of two transformations which maps the curve $y = \ln x$ onto the curve $y = 2\ln(x - 2)$. [3]

(ii) Find the exact coordinates of P and Q. [5]

(iii) Using integration by parts, show that $\int \ln x \, dx = x\ln x - x + c$, where c is an arbitrary constant. [3]

(iv) Hence show that the area of the finite region enclosed by the curve $y = \ln x$, the curve $y = 2\ln(x - 2)$ and the x-axis is $m\ln 2 + n$, where m and n are integers to be determined. [7]

END OF QUESTION PAPER

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